3. Freely adapted from

What If I Double It?
By Thomas Humphrey

Why does size matter?
To find out more, let's cook a turkey.

Suppose you are responsible for cooking a turkey. You have a 9kg turkey, but your cookbook only tells you how long it takes to cook a 4·5kg turkey.

How long do you cook your turkey?
Since the 9kg turkey is twice the size of a 4·5kg bird, at first the answer might seem obvious. Simply double the cooking time suggested for a 4·5kg turkey. But is that really the right thing to do?

The way I see it, there are three ways to answer this question:

(a) you can call and ask your grandmother;
(b) you can get a new cookbook;
(c) you can thumb through your physics books for the turkey equation.

I began by gathering cookbook data. My cookbook says that when you double the weight of a turkey, you don't have to double the cooking time. You only have to increase it from 4 hours for the small bird to 6½ hours for the big one. So even though the 9kg turkey is twice the weight of the 4·5kg turkey, you only have to cook it about 1·6 times as long. Why would that be?

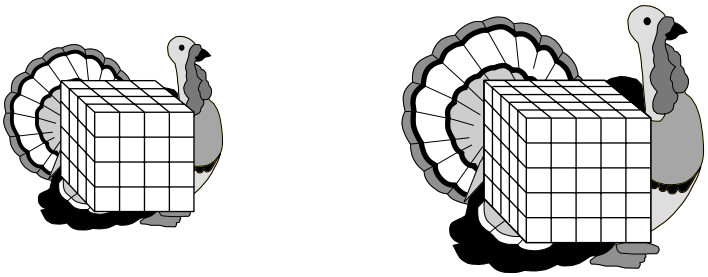
Let's take a more detailed look at our question. What is the "it" that we are doubling? What kind of "its" does a turkey have?

The turkey has a width, a surface area, a volume, and a weight. It has a density, a thermal conductivity (how well it transfers the oven's heat into its interior), and a specific heat capacity (how much heat it needs to climb one degree Celsius in temperature). A turkey has a lot of "its." How do some of these factors change in going from a 4·5kg turkey to a 9kg turkey?

Let's imagine that my turkey is shaped like a cube. This will make it easier to see how the various factors change.

Take a look at the cubical turkeys on page 3. Try to figure out how the weight, surface area, and width differ. If you count the number of small cubes in the 4·5kg turkey, you will find that there are $4 \times 4 \times 4$, or 64 cubes. The number of cubes in the 9kg turkey is $5 \times 5 \times 5$, or 125 cubes. That's not exactly double, but it's pretty close. So now we know that the 9kg cube turkey is about twice the volume of the 4·5kg cube turkey (that is, it has twice as many little cubes), and therefore it weighs about twice as much.

But when you double the size of a turkey, what happens to its width and surface area? Do they double, too?



If you look at the Cube turkeys above, you can see that the widths of the two turkeys are 4 and 5 blocks respectively. So the bigger turkey is about 25 percent wider than the smaller one. It did not double.

If we look at surface area, the small turkey has 6 sides × 16 blocks per side, or 96 blocks. The surface area of the big turkey has 6 sides × 25 blocks per side, or 150 blocks. That means the big turkey has about 50 percent more blocks in it than the small turkey. So that measurement didn't double, either. More precisely, on a real turkey, the width and all the other linear dimensions increase by a factor of 1·26 and the surface area increase by a factor of 1·59.

How do some of these "its" – weight, surface area, and thickness – influence the turkey's cooking time?

Well, first of all, the 9kg turkey, because it has doubled in volume, has twice as much stuff (including stuffing) to heat up, so we need to put twice as much heat into it. Fair enough. How does the heat get in? It is transferred across the surface of the turkey, and it must travel all the way into the centre of the bird. The bigger turkey has more surface. That should speed up the transfer of heat, but the heat must travel a longer way to the centre. That will slow things down. The net result is that it doesn't take twice as long to cook the twice-as-heavy turkey. The physicists agree with the home economist.

Increase in cooking time If you put the three factors together, the cooking time increases by 2 × 0·63 × 1·26 = 1·59. (4 hrs. × 1·59 = 6·4 hrs.) My cook book says to increase the time to 6·5 hours, or by a factor of 1·62. (6·5 hrs. / 4 hrs. = 1·62). That's pretty close!

So now we know how to cook a turkey. But in this little foray into the physics of cooking we discovered that the seemingly innocuous question, "What happens if you double it?" has turned out to be quite complex. We must be very specific about which feature of the turkey we are doubling because we don't seem to be able to double everything at once!

The fact that we cannot double every feature of the turkey at the same time is one example of a very general behaviour in nature, a behaviour that leads to consequences even more important than the difference between a perfect turkey and an overcooked one. When we compare similar objects, one large and one small, not all features of the object are magnified or reduced by the same ratio. This has dramatic consequences for natural behaviour.